Leave blank

**1.** Given $y = x^3 + 4x + 1$, find the value of $\frac{dy}{dx}$ when $x = 3$

**(4)**

$$\frac{dy}{dx} = 3(x^2) + 4(x^0) + 0$$

$$= 3x^2 + 4$$

Let $x = 3$

$$\frac{dy}{dx} = 3(3^2) + 4$$

$$= 27 + 4$$

$$= \boxed{31}$$

Q1

**(Total 4 marks)**

Leave blank

2. Express $\frac{15}{\sqrt{3}} - \sqrt{27}$ in the form $k\sqrt{3}$, where $k$ is an integer.

**(4)**

$\frac{15}{\sqrt{3}}$ : Rationalise the denominator

$$\frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{3}$$

$$= 5\sqrt{3}$$

$\sqrt{27}$ : Simplify

$$= \sqrt{9} \times \sqrt{3}$$

$$= 3\sqrt{3}$$

$$5\sqrt{3} - 3\sqrt{3} = \boxed{2\sqrt{3}} \quad , \quad k = 2$$

---

OR $\frac{15}{\sqrt{3}} - \sqrt{27}$ ; get a common denominator

$$\sqrt{27} = \frac{\sqrt{81}}{\sqrt{3}} \quad , \quad \sqrt{81} = 9$$

$$\frac{15 - 9}{\sqrt{3}} = \frac{6}{\sqrt{3}}$$ Rationalise the denominator

$$\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3}$$

$$= \boxed{2\sqrt{3}} \quad , \quad k = 2$$

Q2

**(Total 4 marks)**



Turn over

Leave blank

**3.** Find

$$\int \left(3x^2 - \frac{4}{x^2}\right) \mathrm{d}x$$

giving each term in its simplest form.

**(4)**

$$3x^2 - \frac{4}{x^2} = 3x^2 - 4x^{-2}$$

$$\int \left(3x^2 - 4x^{-2}\right) dx = 3\frac{x^3}{3} - 4\frac{x^{-1}}{-1} + C$$

$$= \boxed{x^3 + 4x^{-1} + c}$$

$$= \boxed{x^3 + \frac{4}{x} + c}$$

either acceptable

Leave blank

**4.** The line $L_1$ has equation $4x + 2y - 3 = 0$

(a) Find the gradient of $L_1$.

**(2)**

The line $L_2$ is perpendicular to $L_1$ and passes through the point (2, 5).

(b) Find the equation of $L_2$ in the form $y = mx + c$, where $m$ and $c$ are constants.

**(3)**

a)

$2y = 3 - 4x$

$y = \frac{3}{2} - 2x$ make y the subject

$y = \underbrace{-2}_{m}x + \frac{3}{2}$

$y = mx + c$

$m = \boxed{-2}$

OR

use implicit differentiation

$\frac{d}{dx}(4x + 2y - 3) = \frac{d}{dx}0$

$4 + 2\frac{dy}{dx} - 0 = 0$

$2\frac{dy}{dx} = -4$

$\frac{dy}{dx} = \boxed{-2}$

b) gradient of $L_2$ is $\frac{1}{2}$ $\because \frac{1}{2} \times -2 = -1$

posses point (2, 5)

using $y - y_1 = m(x - x_1)$

$y - 5 = \frac{1}{2}(x - 2)$

$y - 5 = \frac{1}{2}x - 1$

$\boxed{y = \frac{1}{2}x + 4}$

Leave blank

**5.** Solve

(a) $2^y = 8$

**(1)**

(b) $2^x \times 4^{x+1} = 8$

**(4)**

a) $\log_2(8) = 3$ this must be done by inspection

$y = \boxed{3}$

b) $4 = 2^2$

$\therefore \; 4^{(x+1)} = (2^2)^{(x+1)}$

$= 2^{2(x+1)}$

$= 2^{2x+2}$

$(x^a)^b = x^{ab}$

e.g $(x^2)^3 = (x^2)(x^2)(x^2)$

$= [(x)(x)][(x)(x)][(x)(x)]$

$= x^6 \quad (6 = 2 \times 3)$

$2^x \times 2^{2x+2} = 2^{x+2x+2}$

$= 2^{3x+2}$

$x^a \times x^b = x^{(a+b)}$

e.g $2^1 \times 2^2 = 2 \times 4$

$= 8$

$= 2^3 \quad (3 = 1+2)$

$\therefore \; 2^{3x+2} = 8$

From part (a) we know $2^y = 8$ then $y = 3$

Let $3x+2 = y$

$2^y = 8 \Rightarrow 3x+2 = 3$

$3x = 1$

$\boxed{x = \frac{1}{3}}$

Leave blank

**6.** A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1$$

$$x_{n+1} = (x_n)^2 - kx_n, \quad n \geqslant 1$$

where $k$ is a constant, $k \neq 0$

(a) Find an expression for $x_2$ in terms of $k$. **(1)**

(b) Show that $x_3 = 1 - 3k + 2k^2$ **(2)**

Given also that $x_3 = 1$,

(c) calculate the value of $k$. **(3)**

(d) Hence find the value of $\sum_{n=1}^{100} x_n$ **(3)**

a) $n=1$

$$x_2 = (x_1)^2 - kx_1$$

$$= 1^2 - k$$

$$= \boxed{1-k}$$

b) $n=2$

$$x_3 = (x_2)^2 - k(x_2)$$

$$x_3 = (1-k)^2 - k(1-k)$$

| | 1 | $-k$ |
|---|---|---|
| 1 | 1 | $-k$ |
| $-k$ | $-k$ | $k^2$ |

| $\times$ | 1 | $-k$ |
|---|---|---|
| $-k$ | $-k$ | $+k$ |

$$= k^2 - 2k + 1 - k + k^2$$

$$= 2k^2 - 3k + 1$$

$$= \boxed{1 - 3k + 2k^2}$$ (asked for in question)



P 4 2 8 2 3 A 0 1 0 3 2

Leave blank

**Question 6 continued**

c) given $x_3 = 1$

$1 = 1 - 3k + 2k^2$

Do not Divide by k
this MUST be solved as a qudratic to get the marks

$0 = -3k + 2k^2$

$0 = k(2k - 3)$

either $k = 0$ (question states $k \neq 0$ so ignore this)

OR $2k - 3 = 0$

$2k = 3$

$k = \frac{3}{2}$

d)

$x_2 = 1 - \frac{3}{2}$

$x_2 = -\frac{1}{2}$

we know $x_1 = 1$ & $x_3 = 1$

$\therefore$ this function is cyclical

$\therefore \sum_{n=1}^{n} x_n = 1 + (\frac{1}{2}) + 1 + (-\frac{1}{2}) + 1 \ldots$

$= (\frac{1}{2}) \times \frac{n}{2}$ assuming $\frac{n}{2} \in \mathbb{Z}^+$

$n = 100 \therefore \frac{n}{2} = 50$ , $\frac{1}{2} \times 50 = \boxed{25}$

Leave blank

7. Each year, Abbie pays into a savings scheme. In the first year she pays in £500. Her payments then increase by £200 each year so that she pays £700 in the second year, £900 in the third year and so on.

(a) Find out how much Abbie pays into the savings scheme in the tenth year. **(2)**

Abbie pays into the scheme for $n$ years until she has paid in a total of £67 200.

(b) Show that $n^2 + 4n - 24 \times 28 = 0$ **(5)**

(c) Hence find the number of years that Abbie pays into the savings scheme. **(2)**

a) $U_n = a + (n-1)d$ Quote the formula for Method Marks

$a = £500,\ n = 10,\ d = £200$

$U_{10} = 500 + 9(200)$

$U_{10} = \boxed{£2300}$

b) $S_n = \frac{1}{2}n[2a + (n-1)d]$

$S_n = £67{,}200,\ a = £500,\ d = £200$

$67{,}200 = 500n + 100n(n-1)$ Plug in Values

$67200 = 500n + 100n^2 - 100n$ expand

$100n^2 + 400n - 67{,}200 = 0$ Rearrange

$n^2 + 4n - 672 = 0$ $0 \div 100 = 0$

$24 \times 28 = 500 + 170 + 2 = 672$ $672 = 24 \times 28$

$\boxed{n^2 + 4n - (24 \times 28) = 0}$ question format

Leave blank

**Question 7 continued**

c) $n^2 + 4n - (24 \times 28) = 0$

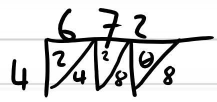
$n(n+4) = 24 \times 28$

By inspection $\boxed{n = 24}$

When the question is in a suspicious format (like this) then assume that it is a hint. Since this is a non-calculator paper, trying to complete this using the quadratic formula may be time consuming and requires a lot of trial an error.

In the exam, if you have to use the quadratic formula, here is a tip for finding 3/4 digit square roots.

$n^2 + 4n - 672 = 0$



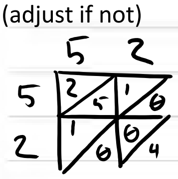
$$\frac{-4 \pm \sqrt{16 - -2688}}{2} = \frac{-4 \pm \sqrt{2704}}{2}$$

Spit the number into 2 2 digit numbers: 27, 04

Find the largest square number equal to or less than them: 25, 04

Square root them and use them as a 2 digit number: 5, 2

Square the number to see if you're right (adjust if not): 52



$52^2 = 2,704$

$$\frac{-4 \pm 52}{2} = 24, -28$$

Be warned though, this is time consuming and won't always work (say if the number isn't a square number)
Only use this as a last resort once you have finished all other questions

Leave blank

**8.** A rectangular room has a width of $x$ m.

The length of the room is 4 m longer than its width.

Given that the perimeter of the room is greater than 19.2 m,

(a) show that $x > 2.8$

**(3)**

Given also that the area of the room is less than 21 m$^2$,

(b) (i) write down an inequality, in terms of $x$, for the area of the room.

(ii) Solve this inequality.

**(4)**

(c) Hence find the range of possible values for $x$.

**(1)**

a) width (w) $= x$ Length (L) $= x+4$

$2w + 2L =$ Perimeter

$2x + 2(x+4) > 19.6$

$4x + 8 > 19.6$

$4x > 11.6$

$\boxed{x > 2.8}$

b) i) Area $= L \times W$

$= x(x+4)$

given $x^2 + 4x < 21$

$\boxed{x^2 + 4x - 21 < 0}$

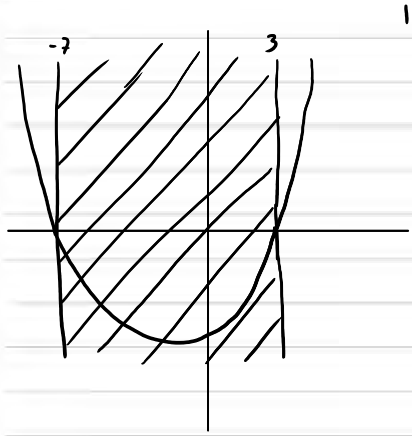
ii) $(x-3)(x+7) < 0$

| $\times$ | $x$ | $-3$ |
|---|---|---|
| $x$ | $x^2$ | $-3x$ |
| $7$ | $7x$ | $-21$ |

Leave blank

**Question 8 continued**

ii)




$-7 < x < 3$

c) $-7 < x < 3$ and $2.8 < x$

$\therefore \boxed{2.8 < x < 3}$

Leave blank

**9.**

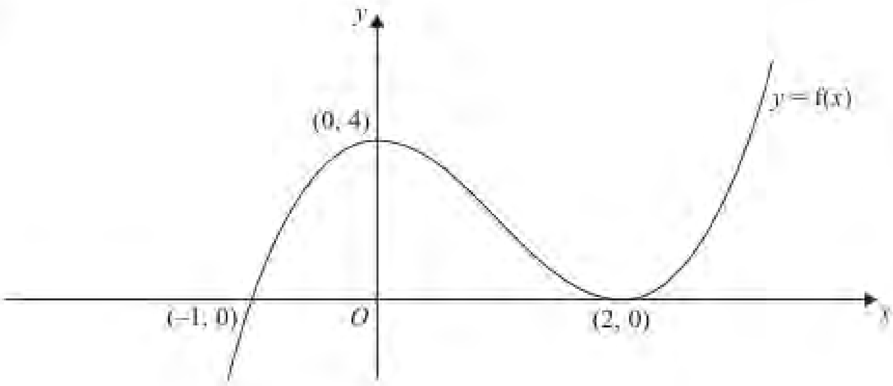



**Figure 1**

Figure 1 shows a sketch of the curve $C$ with equation $y = f(x)$.

The curve $C$ passes through the point $(-1, 0)$ and touches the $x$-axis at the point $(2, 0)$.

The curve $C$ has a maximum at the point $(0, 4)$.

(a) The equation of the curve $C$ can be written in the form

$$y = x^3 + ax^2 + bx + c$$

where $a$, $b$ and $c$ are integers.

Calculate the values of $a$, $b$ and $c$.

**(5)**

(b) Sketch the curve with equation $y = f(\frac{1}{2}x)$ in the space provided on page 24

Show clearly the coordinates of all the points where the curve crosses or meets the coordinate axes.

**(3)**

a)

cuts $x$ axis at $x=-1$, $x=2$ & $x=2$ turning point

$\therefore y = (x+1)(x-2)^2$

| x | $x$ | $-2$ |
|---|---|---|
| $x$ | $x^2$ | $-2x$ |
| $-2$ | $-2x$ | $+4$ |

| x | $x^2$ | $-4x$ | $+4$ |
|---|---|---|---|
| $x$ | $x^3$ | $-4x^2$ | $+4x$ |
| $1$ | $x^2$ | $-4x$ | $+4$ |

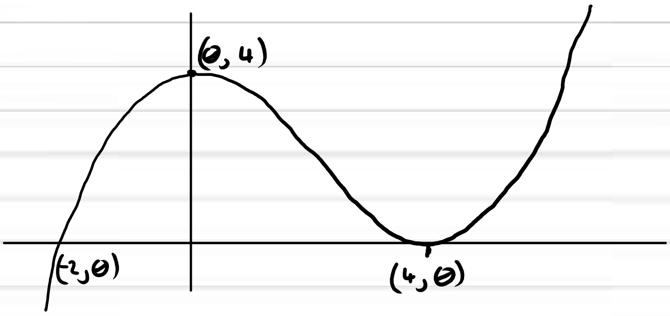
$$\boxed{\begin{aligned} a &= 3 \\ b &= 0 \\ c &= 4 \end{aligned}}$$

$y = x^3 - 3x^2 + 0x + 4$

Leave blank

**Question 9 continued**

b)






Turn over

P 4 2 8 2 3 A 0 2 3 3 2

PMT

Leave blank

**10.** A curve has equation $y = \mathrm{f}(x)$. The point $P$ with coordinates $(9, 0)$ lies on the curve.

Given that

$$\mathrm{f}'(x) = \frac{x+9}{\sqrt{x}}, \qquad x > 0$$

(a) find $\mathrm{f}(x)$.

**(6)**

(b) Find the $x$-coordinates of the two points on $y = \mathrm{f}(x)$ where the gradient of the curve is equal to 10

**(4)**

a) $f'(x) = \frac{x}{\sqrt{x}} + \frac{9}{\sqrt{x}}$

$= x^{\frac{1}{2}} + 9x^{-\frac{1}{2}}$ Use exponentials

$\int f'(x)\,dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 9\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$f(x) = \frac{2}{3}x^{\frac{3}{2}} + 18x^{\frac{1}{2}} + c$

given $f(9) = 0$ Remember to find the $+c$

$\frac{2}{3}(9^{\frac{3}{2}}) + 18(9^{\frac{1}{2}}) + c = 0$ $9^{\frac{3}{2}} = \sqrt{9}^{3}$

$\frac{2}{3}(3^3) + 18(3) + c = 0$

$18 + 54 + c = 0$

$c = -72$

$f(x) = \boxed{\frac{2}{3}x^{\frac{3}{2}} + 18x^{\frac{1}{2}} - 72}$



P 4 2 8 2 3 A 0 2 6 3 2

Leave blank

**10.** A curve has equation $y = \mathrm{f}(x)$. The point $P$ with coordinates $(9, 0)$ lies on the curve.

Given that

$$\mathrm{f}'(x) = \frac{x+9}{\sqrt{x}}, \qquad x > 0$$

~~(a) Find f(x).~~

**(6)**

(b) Find the $x$-coordinates of the two points on $y = \mathrm{f}(x)$ where the gradient of the curve is equal to 10

**(4)**

b) $f'(x) = 10$

$$\frac{x+9}{\sqrt{x}} = 10$$

let $y = \sqrt{x}$

so $y^2 = x$

$y^2 + 9 = 10y$

$y^2 - 10y + 9 = 0$

| $\times$ | $y$ | $-9$ |
|---|---|---|
| $y$ | $y^2$ | $-9y$ |
| $-1$ | $-y$ | $9$ |

$(y-9)(y-1) = 0$

$\sqrt{x} = 9$ or $\sqrt{x} = 1$

$\boxed{x = 81 \text{ or } x = 1}$



P 4 2 8 2 3 A 0 2 6 3 2

Leave blank

**11.**

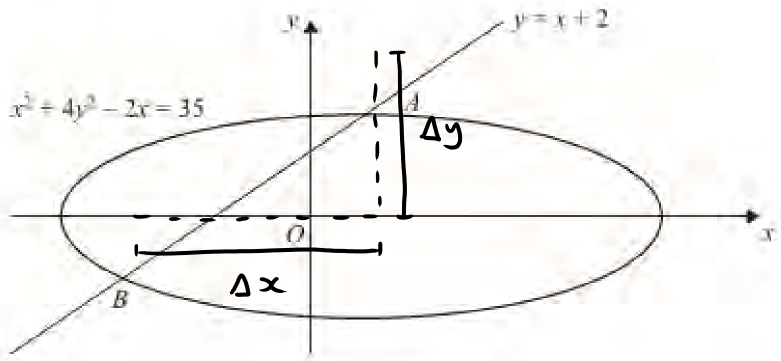



**Figure 2**

The line $y = x + 2$ meets the curve $x^2 + 4y^2 - 2x = 35$ at the points $A$ and $B$ as shown in Figure 2.

(a) Find the coordinates of $A$ and the coordinates of $B$.
**(6)**

(b) Find the distance $AB$ in the form $r\sqrt{2}$ where $r$ is a rational number.
**(3)**

$y = x+2$

$x^2 + 4y^2 - 2x = 35$

$y^2 = x^2 + 4x + 4$

| x | x | +2 |
|---|---|---|
| x | $x^2$ | $2x$ |
| +2 | $2x$ | 4 |

$x^2 + 4(x^2+4x+4) - 2x - 35 = 0$

$x^2 + 4x^2 + 16x + 16 - 2x - 35 = 0$

$5x^2 + 14x - 19 = 0$

| x | x | -1 |
|---|---|---|
| 5x | $x^2$ | $-5x$ |
| +19 | $19x$ | $-19$ |

$(5x+19)(x-1) = 0$

$x = 1, \quad x = -\frac{19}{5}$

Since $y = x+2$

$y = 3, \quad y = -\frac{9}{5}$

$\boxed{(1,3) \ \& \ (-\frac{19}{5}, -\frac{9}{5})}$



P 4 2 8 2 3 A 0 3 0 3 2

Leave blank

**Question 11 continued**

b)

$\Delta x = 1 - -\frac{19}{5}$ see graph

$= \frac{24}{5}$

$\Delta y = 3 - -\frac{9}{5}$

$= \frac{24}{5}$

$\sqrt{(\Delta x)^2 + (\Delta y)^2} = |AB|$ Pythagoras' theorem

$\sqrt{\left(\frac{24}{5}\right)^2 + \left(\frac{24}{5}\right)^2} = \sqrt{2\left(\frac{24}{5}\right)^2}$

$= \boxed{\sqrt{2}\left(\frac{24}{5}\right)}$

Q11

**(Total 9 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**